Solutions

Chapter 4
Exponential Word Problems

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1. Helen invests \$10,000 in a high-yield uninsured certificate of deposit that pays 12% interest annually, compounded every 6 months.

(a) Find a formula for the amount A of the certificate after t years.

(b) What is the amount after 3 years?

(c) How long will it take for her investment to grow up to \$25,000?

(a) $A(t) = 10{,}000\left(1+\frac{.12}{2}\right)^{2t}$

(b) $A(3) = 10{,}000(1.06)^6 \approx \$14{,}185.19$

(c) $25{,}000 = 10{,}000(1.06)^{2t}$

$2.5 = (1.06)^{2t} \Rightarrow \log(2.5) = 2t\log(1.06)$

$t \approx 7.86$ years

2. On January 1, 2007, the population of the world was approximately 6.6 billion and was increasing by 1.36% every year. Assume that this rate of increase continues.

(a) Find a function P that models the population after t years.

(b) What does the model predict for the population in 2015?

(c) In what year will the population of the world have doubled?

(a) $P(t) = 6.6(1.0136)^t$

(b) $P(8) = 6.6(1.0136)^8 \approx 7.35$ billion people

(c) $13.2 = 6.6(1.0136)^t$

$2 = 1.0136^t$

$\log(2) = t\log(1.0136) \Rightarrow t \approx 51.31$ years.

Year 2,064

Newton's Law of Cooling: When a hot object, such as a cup of coffee, is left to cool, its temperature decreases continuously at a rate proportional to the temperature difference between the object and its surroundings. Since this difference is continually changing, the rate at which the object cools is continuously changing. Newton's Law of Cooling states the the temperature T of a cooling object is modeled by

$$T = A + (I - A)e^{-kt}$$

where t is time since the object began cooling, I is the initial temperature of the object, A is the ambient temperature (the temperature of the surroundings), and k is a constant which depends on the type of object.

3. Crime Scene Investigation: Police officers arrive at a crime scene and find a tub full of warm water. A thermometer shows that the water temperature is 76°F and the air temperature is 70°F. It is known that most people fill a tub with water at 100°F.

(a) Use Newton's Law of Cooling to model the temperature of the water in the tub. (Measure t in minutes and use the heat transfer coefficient $k = 0.018$.)

(b) How long has the bathtub been cooling?

(a) $T = 70 + (100-70) \cdot e^{-0.018t}$

$= 70 + 30 \cdot e^{-0.018t}$

(b) $76 = 70 + 30 \cdot e^{-0.018t}$

$6 = 30 \cdot e^{-0.018t}$

$1/5 = e^{-0.018t}$

$\ln(1/5) = -0.018t$

$t \approx 89.41$ minutes

4. Dating the Iceman: On September 19, 1991, Erika and Helmut Simon were hiking in the Alps near the Aurstrian-Italian border. As they approached an ice-filled depression, they were surprised to see the frozen body of a man sticking halfway out of the ice. After the authorities arrived, they noticed he had goatskin leather clothing, a bronze axe and a quiver of arrows. Tissues samples from the iceman indicated that his body had 57.67% of the carbon-14 that is present in a living person. Estimate how long ago the iceman died (in years) using the formula

$$m(t) = C(\frac{1}{2})^{t/h}$$

where C is the initial mass of the radioactive substance, h is its half-life, and $m(t)$ is the mass remaining at time t.

Half-life of Carbon-14 is 5730 years

$$0.5767 = \left(\frac{1}{2}\right)^{t/5730}$$

$$\ln(0.5767) = \frac{t}{5730} \ln(1/2)$$

$$t = \frac{5730 \cdot \ln(0.5767)}{\ln(1/2)} \approx 4{,}550 \text{ years.}$$

5. Masako is planning to invest \$5000 in a certificate of deposit. How long does it take for the investment to grow to \$8000 under the given conditions?

(a) The certificate of deposit pays 3.55% interest annually, compounded every month.

(b) The certificate of deposit pays 3.05% interest annually, compounded continuously.

(a) $8000 = 5000\left(1.\cancel{07} + \frac{.0355}{12}\right)^{12t}$

$\ln(8/5) = 12t \ln(1.002958) \Rightarrow t \approx 13.26 \text{ years}$

(b) $8000 = 5000\, e^{.0305t}$

$\ln(8/5) = .0305t \Rightarrow t \approx 15.41 \text{ years}$

6. The half-life of radium-226 is 1600 years. Suppose we have a 22-mg sample.

(a) Find the yearly growth factor a.

(b) Find an exponential model $m(t) = Ca^t$ for the mass remaining after t years.

(c) How much of the sample will remain after 4000 years?

(d) After how long will only 18mg of the sample remain?

(a) $a = \left(\frac{1}{2}\right)^{1/1600}$

(b) $m(t) = 22 \cdot \left(\frac{1}{2}\right)^{t/1600}$

(c) $m(4000) = 22 \cdot \left(\frac{1}{2}\right)^{4000/1600} \approx 3.89 \text{ mg}$

(d) $18 = 22 \cdot \left(\frac{1}{2}\right)^{t/1600}$

$\ln\left(\frac{9}{11}\right) = \frac{t}{1600} \ln\left(\frac{1}{2}\right) \Rightarrow t = \frac{1600 \ln(9/11)}{\ln(1/2)} \approx 463 \text{ years}$